NAME:______________________

ECEN 5713 System Theory
Spring 1997
Midterm Exam #3

I, ____________________, promise that I won't seek any help from others. And I won't discuss with anyone else. _________________________ ___________

signature ***date***

<u>Vector Space & Linear Algebra</u> (choose any two, 20%)

<u>Problem 1a</u>) Find a basis for the space X generated by the vectors

$$\begin{bmatrix} 1 \\ 3 \\ -7 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}.$$

And what is the dimension of X ?

<u>Problem 1b</u>) Is the following set of vectors linear independent

i) in $(\Re(s), \Re)$, and

ii) in $(\Re(s), \Re(s))$?

$$\left\{ \frac{3s^2 - 12}{2s^3 + 4s - 1}, \frac{4s^5 + s^3 - 2s - 1}{1}, \frac{1}{s^2 + s - 1} \right\}$$

<u>Problem 1c</u>) Find an orthonormal basis from vectors

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

by Gram-Schmidt or modified Gram-Schmidt procedures.

<u>Problem 1d</u>) Consider linear operator

$$A = \begin{bmatrix} 1 & 4 & 7 & 3 \\ 2 & 0 & 2 & 1 \\ 3 & 4 & 9 & 4 \end{bmatrix},$$

Find its rank, nullity, range space and null space.

<u>Eigenvalues & Eigenvectors</u> (20%)

<u>Problem 2a</u>) Find the eigenvalues, eigenvectors and Jordan form for the matrix

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$$

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Problem 2b) A 6×6 matrix A has characteristic polynomial $\Delta(\lambda) = (\lambda - 1)^4 (\lambda - 2)^2 = 0$. One possible Jordan-form representation is

$$\overline{A} = \begin{bmatrix} 1 & 1 & & & & \\ & 1 & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & 2 & 1 \\ & & & & & 2 \end{bmatrix}.$$

Please determine its structure (i.e., rank and nullity of $(A - \lambda_i I)^k$).

Fundamental Matrix & State Transition Matrix (20%)

Problem 3a) Find the fundamental matrix of the homogeneous equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} t & 0 & 0 \\ 1 & t & 0 \\ e^{-t} & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Problem 3b) Show directly from the Peano-Baker series that if $A(t) = A$ then

$$\Phi(t, t_0) = \exp A(t - t_0).$$

Solution of Dynamic System (20%)

Problem 4a) Given $\dot{x} = t^2 Ax$ where A is an $n \times n$ real matrix, determine $x(t)$ in terms of A and $x(t_0)$.

Problem 4b) Consider $x(k+1) = A(k)x(k)$. Define

$$\begin{aligned} &\Phi(k,m) = A(k-1)A(k-2)\cdots A(m), \quad \text{for } k > m \\ &\Phi(m,m) = I \end{aligned}.$$

Show that, given the initial state $x(m) = x_0$, the state at iteration k is given by $x(k) = \Phi(k,m)x_0$. If A is independent of k, what is $\Phi(k,m)$?

Function of At (20%)

Problem 5a) If $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$, show that

$$e^{At} = \begin{bmatrix} e^{\sigma t} \cos \omega t & e^{\sigma t} \sin \omega t \\ -e^{\sigma t} \sin \omega t & e^{\sigma t} \cos \omega t \end{bmatrix}.$$

Problem 5b) Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$, find A^{101}, e^{At} and $\sin(At)$.

HOW LONG YOU HAVE SPENT ON THIS EXAM ?